Wavelet Analysis Status Report
May 20, 2005

Summary

Sample data were analyzed with two wavelet basis functions. Using a Morlet wavelet transform, in all data sets there is clear evidence of signals with λ between 20 and 40 s.

All data sets were analyzed with two wavelet functions: derivative of a Gaussian (DOG), order $m=2$; and Morlet, $\omega_0=6$. The wavelet functions are for DOG

$$\psi_0 = \frac{-1}{\sqrt{\Gamma(5/2)}}(\eta^2 - 1)(e^{-\eta^2/2})$$

and for Morlet

$$\psi_0 = \pi^{-1/4} e^{i\omega_0 \eta} e^{-\eta^2/2}.$$

The DOG wavelet is better suited to identifying transient events in a time series of data, while the Morlet wavelet is better suited to identifying the frequency components present. (The DOG wavelet is narrower in the time domain and broader in the spectral domain than the Morlet.)

Figures 1 through 9 show wavelet scalograms for nine data sets: the first five data sets are those provided, and the last four are test cases to illustrate the wavelet analysis technique. the first test case has two impulses, one of short duration and one of longer duration. The second test case is a cosine function (amplitude 2.0) with two cycles over the data period. The third test case is the same cosine function but with Gaussian white noise added ($\sigma=2.0$). The final test case is just Gaussian white noise, generated with the same random number seed as for the noise added to the cosine function.

In each figure, the upper portion is the DOG transform, the lower the Morlet. In each part there is a small plot of the data for reference, in every case the mean of the data set has been subtracted. The scalograms are 2-dimensional plots where the x axis corresponds to the data set time, the y axis is the effective Fourier wavelength of the wavelet basis (a factor of ~1 relates the wavelet scale s to the Fourier wavelength for the Morlet transformation, a factor of ~4 for the DOG basis), and the color represents the wavelet power spectrum. The color scale corresponds to a normalized amplitude of the wavelet transform $W_n(s)$:

$$W_n(s) = \sum_{n'=0}^{N-1} x_{n'} \sqrt{\frac{\delta t}{s}} \psi_0\left(\frac{(n'-n)\delta t}{s}\right),$$

where n is the time location of the transform (i.e. the x-axis of the scalogram), s is the wavelet scale (the y axis of the scalogram), n' is the index for the sum over the N data points, and δt is the time spacing of the data. The normalization used for the two bases is as follows: for the DOG transform the color scale represents $W_n(s)/\sigma^2$, as the transform is real valued; and for the

Morlet transform the color scale represents $|W_n(s)|^2/\sigma^2$. A value of 9.0 for the normalized Morlet transform corresponds to a 3-σ effect, given that the variance of the data is representative of the noise level.

A brief summary of some characteristics of the data follows in Table 1.

Data Set Number	Name	σ of the Set	Morlet Maximum	Location
0	MCDR2_run10	0.247	7.3	n=20 s=41
1	MCDR4_run10	0.061	11.1	n=24, s=30
2	MCDR5_run10	0.247	7.01	n=20, s=30
3	MCDR6_run10	0.760	11.3	n=18, s=22
4	MCDR10_run20	1.021	13.4	n=24, s=20
5	Test Impulse	0.758	5.0	n=30, s=21
6	Test cosine	1.414	20.4	n=23, s=22
7	Test cosine +noise	2.586	7.7	n=18, s=22
8	Test noise	2.163	4.7	n=31, s=2

Table 1. Summary of data sets.

Comparison of all of the sample data to the test data cases shows evidence for events in each of the sample sets. The Morlet transform appears to provide strong evidence for features in each of the five sets. The s value of the location of the Morlet maximum given in Table 1 is the effective Fourier wavelength of the identified signal. A brief discussion of each data set follows. [Note that normalizing the sample data sets by a common value (rather than by the individual variance of each set) would allow comparison of the relative wavelet power spectra. Here the transforms were individually normalized to allow for identification of a signal in each set with some level of statistical understanding. The Morlet maximum and σ values in Table 1 allow for comparisons of this sort to be made.]

Data Set 0 This set appears to have two events or signals within the time series. A signal with wavelength 41 seconds appears throughout, while a faster signal ($\lambda = 18$ s) is present at the beginning of the set.

Data Set 1 There is clear evidence for a signal with $\lambda = 30$ s. The wavelet transform feature at s = 10 seconds or so is either a very weak signal or an artifact of the data.

Data Set 2 This set appears to have two separate events, or possibly one event with a shifting frequency. In that case the DOG transform illustrates the shift in frequency with time, the peaks/valleys (yellow/purple/yellow/purple) shifting to lower s value as the data progresses.

Data Sets 3 and 4 Clear evidence for a signal with $\lambda = 20$ to 22 s.

Data Sets 5 and 6 These test data show that the DOG basis is good at localizing events in time (Set 5) and the Morlet good at localizing events in frequency space (Set 6).

Data Set 7 Introducing white noise (σ=2.0) into the cosine signal does not affect the Morlet transform peak area other than to reduce its amplitude. The cosine is not obvious from casual inspection of the data, but it is clearly evident in the transform.

Data Set 8 This set is noise only, and the same noise as was added to Set 6 to produce Set 7. Note the same features at the lower part of the Morlet transform plots for this set and for Set 7, but the absence here of the peak from the cosine signal.
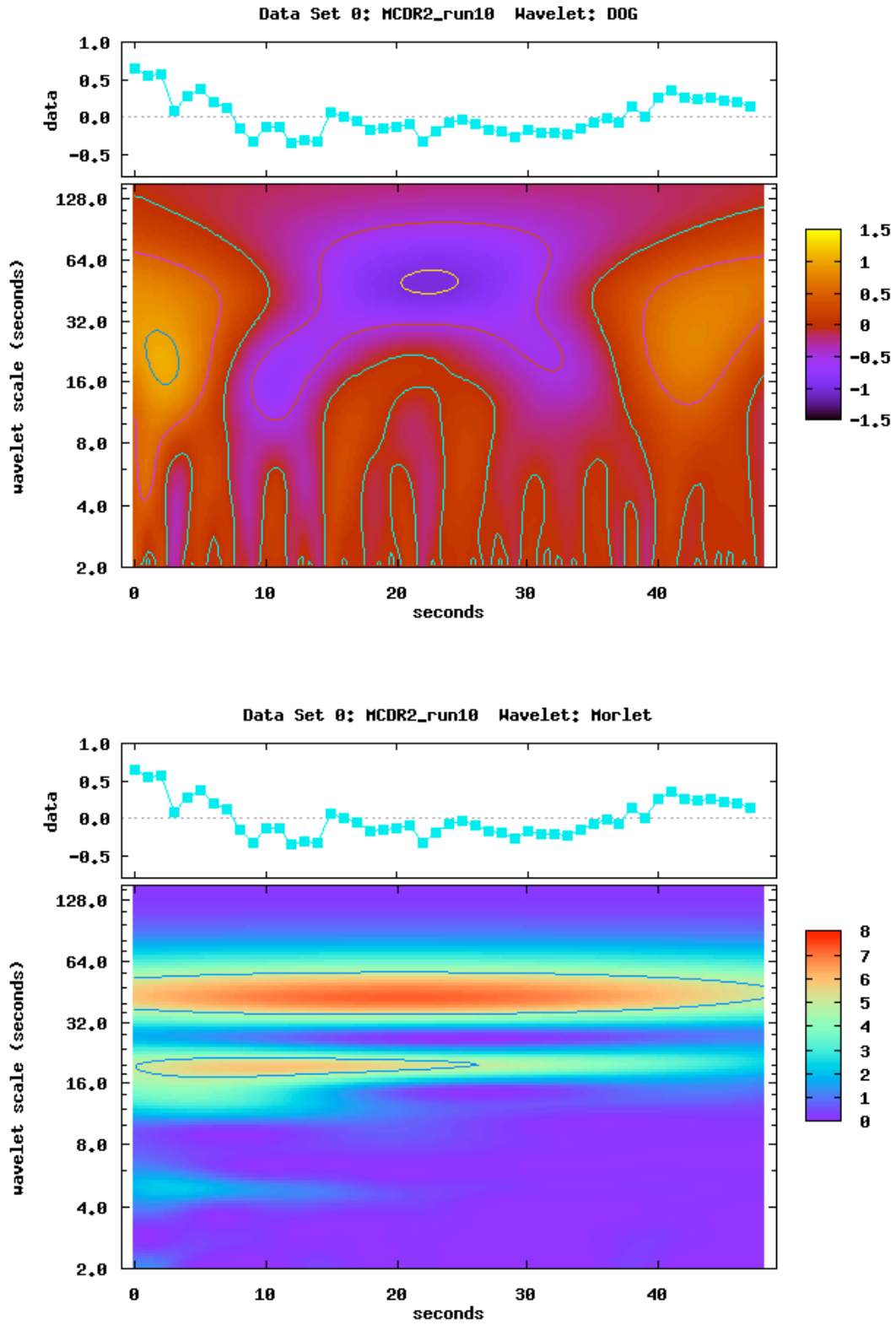
Figure 1. Wavelet transform scalograms for the MCDR2_run10 data set.

Data Set 1: MCDR4_run10   Wavelet: DOG

Data Set 1: MCDR4_run10   Wavelet: Morlet
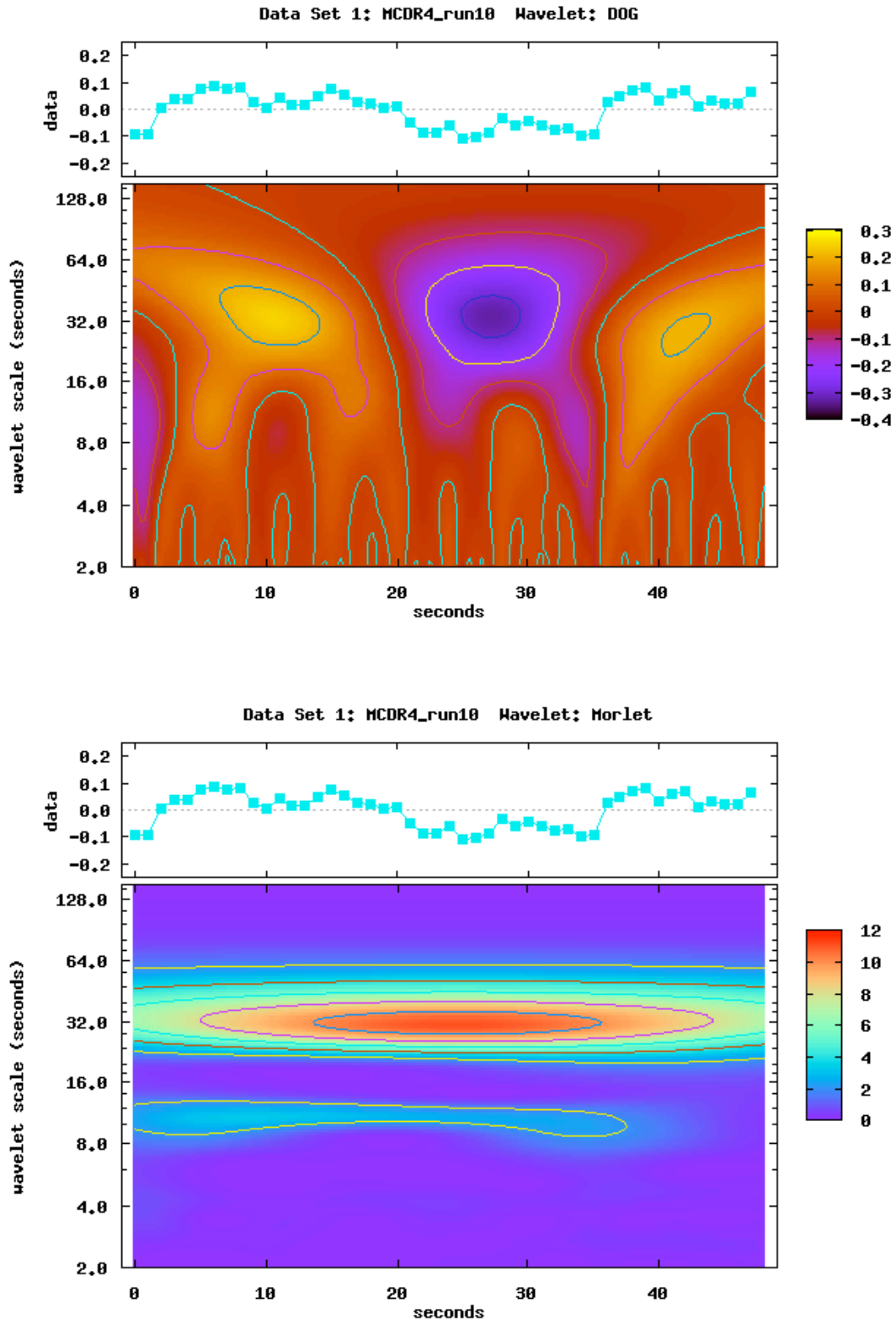
Figure 2. Wavelet transform scalograms for the MCDR4_run10 data set.
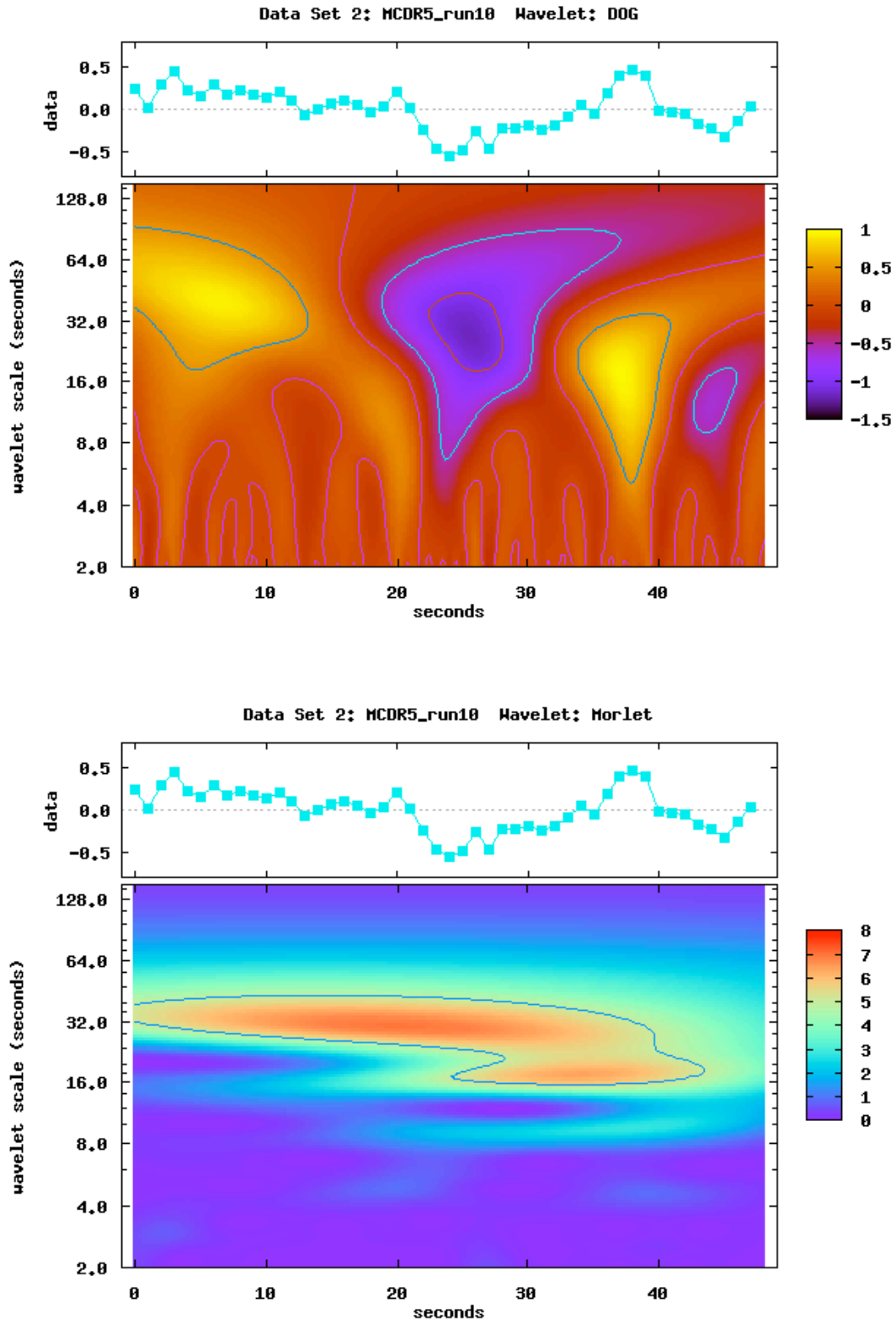
Figure 3. Wavelet transform scalograms for the MCDR5_run10 data set.

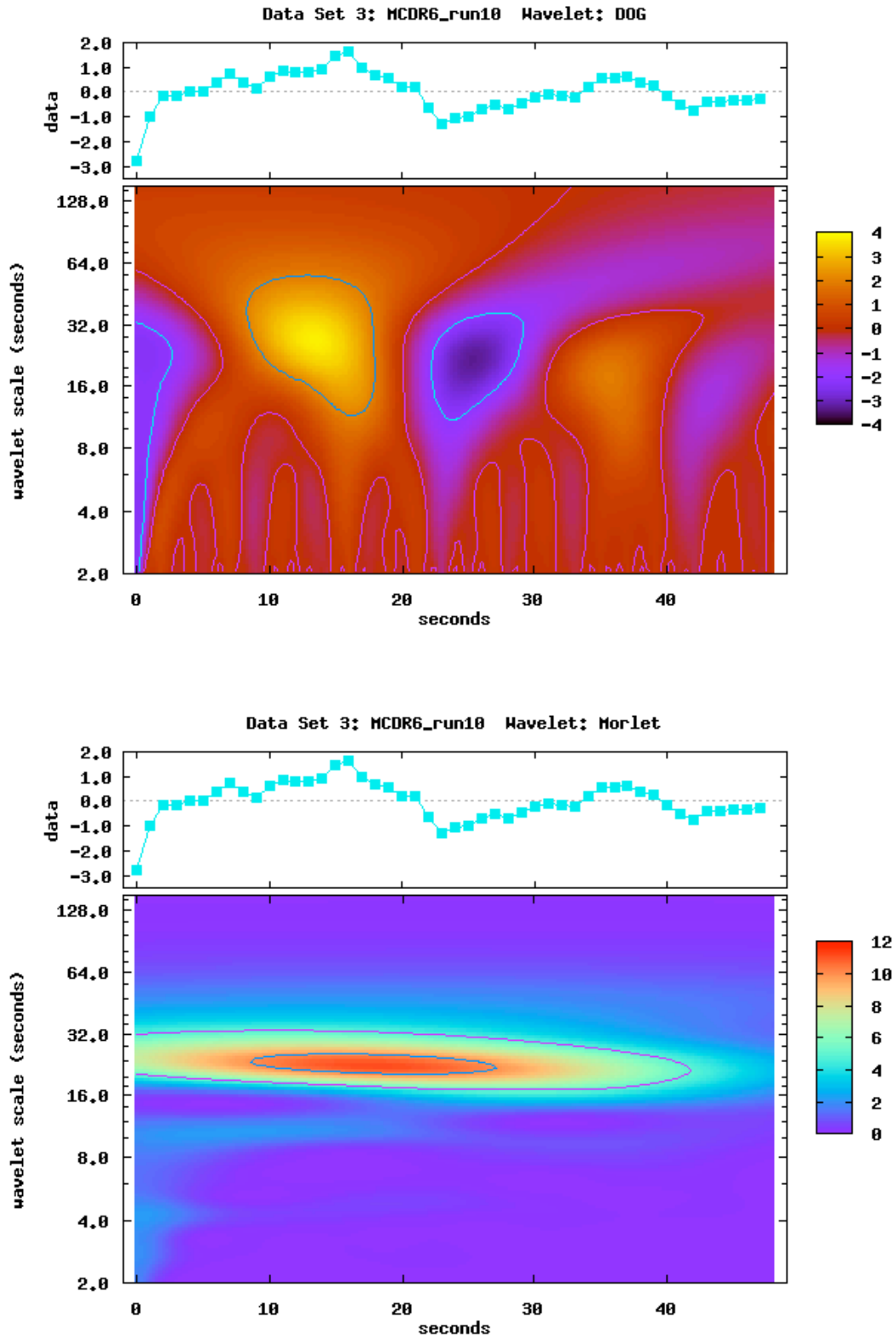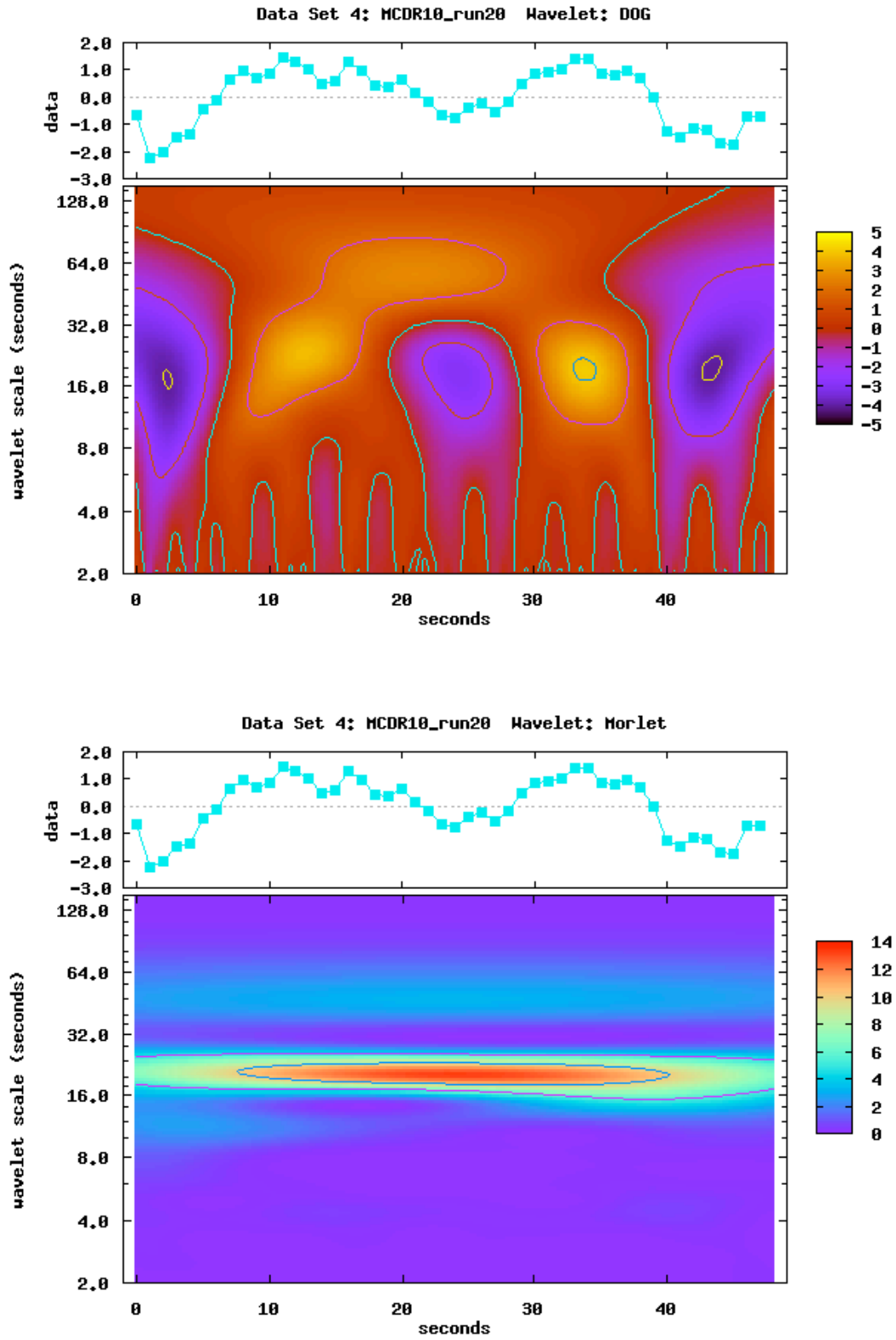Figure 4. Wavelet transform scalograms for the MCDR6_run10 data set.

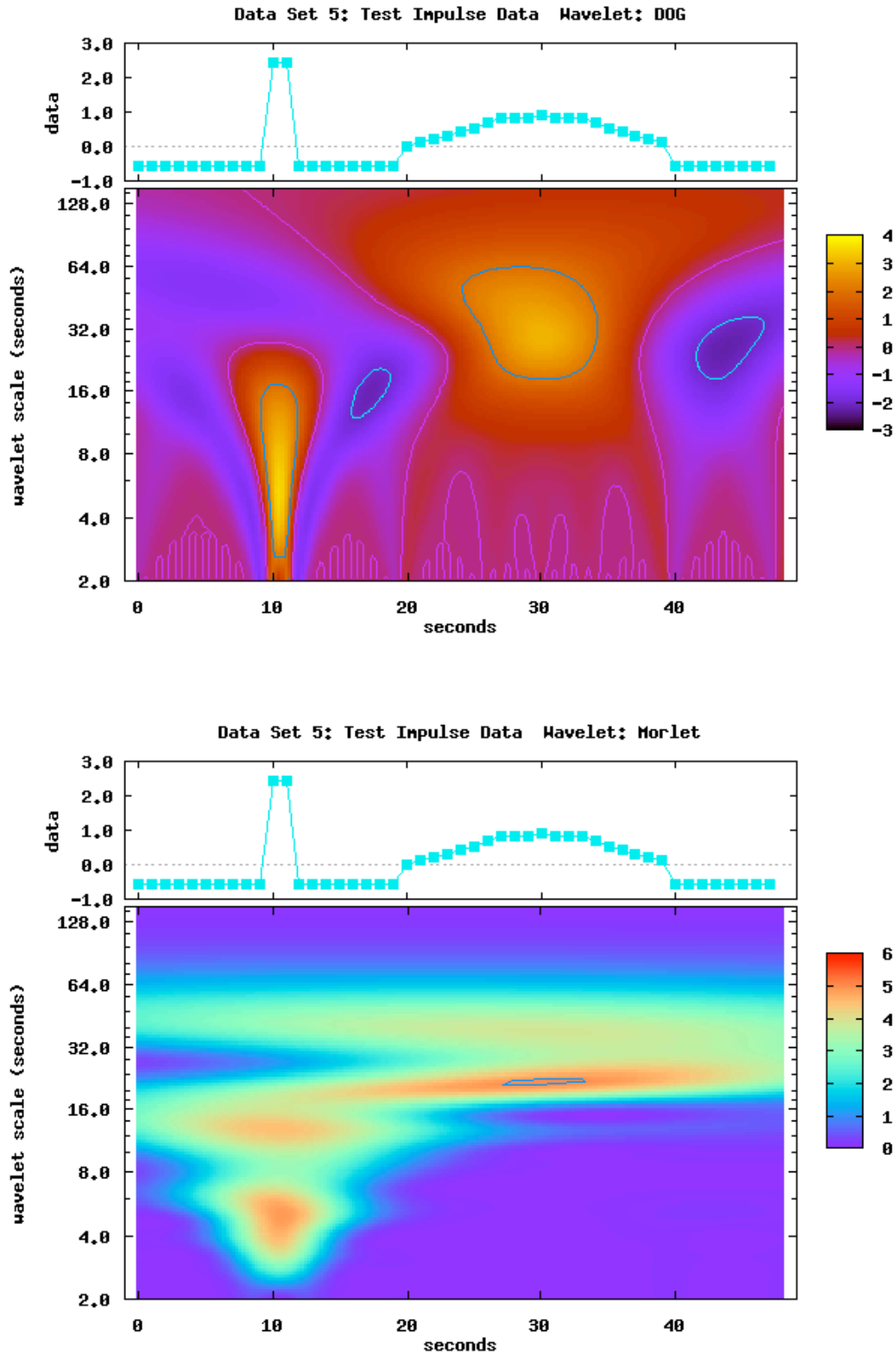Figure 5. Wavelet transform scalograms for the MCDR10_run20 data set.

Data Set 5: Test Impulse Data   Wavelet: DOG

Data Set 5: Test Impulse Data   Wavelet: Morlet

Figure 6. Wavelet transform scalograms for a test data set with two impulses.

Data Set 6: Test Cosine   Wavelet: DOG

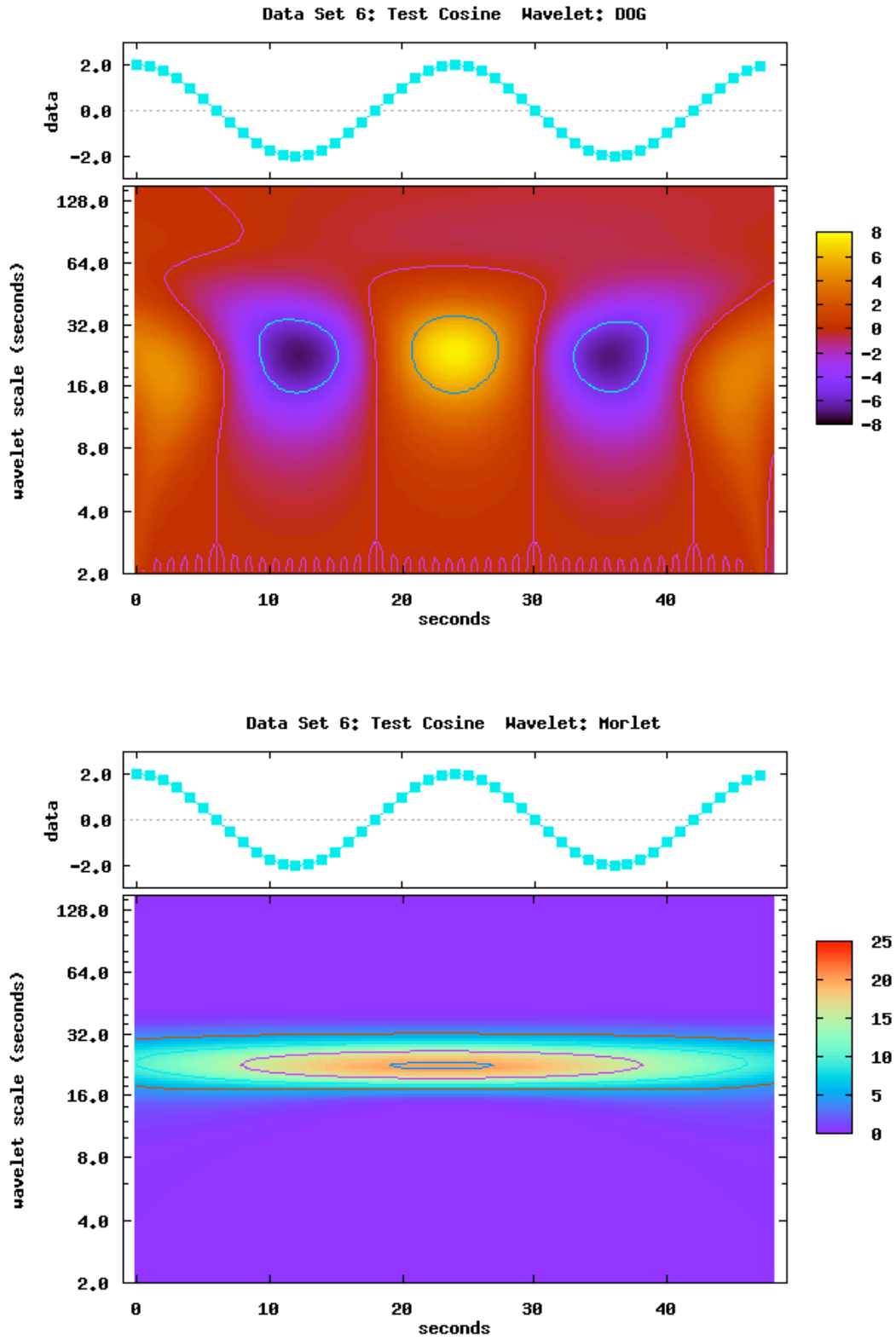Data Set 6: Test Cosine   Wavelet: Morlet

Figure 7. Wavelet transform scalograms for a cosine function.
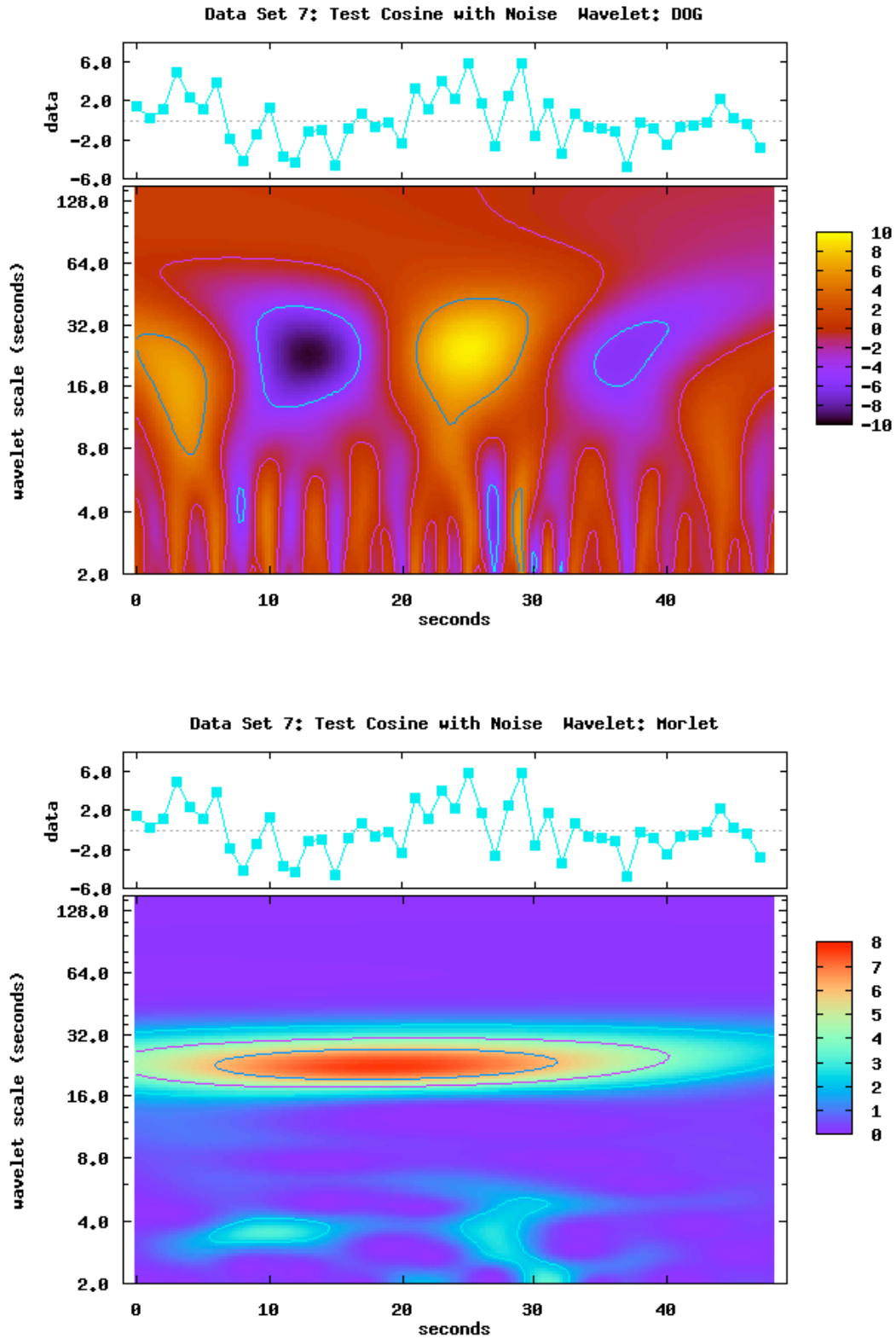
Figure 8. Wavelet transform scalograms for a cosine function (amplitude 2.0) with Gaussian noise added ($\sigma = 2.0$).
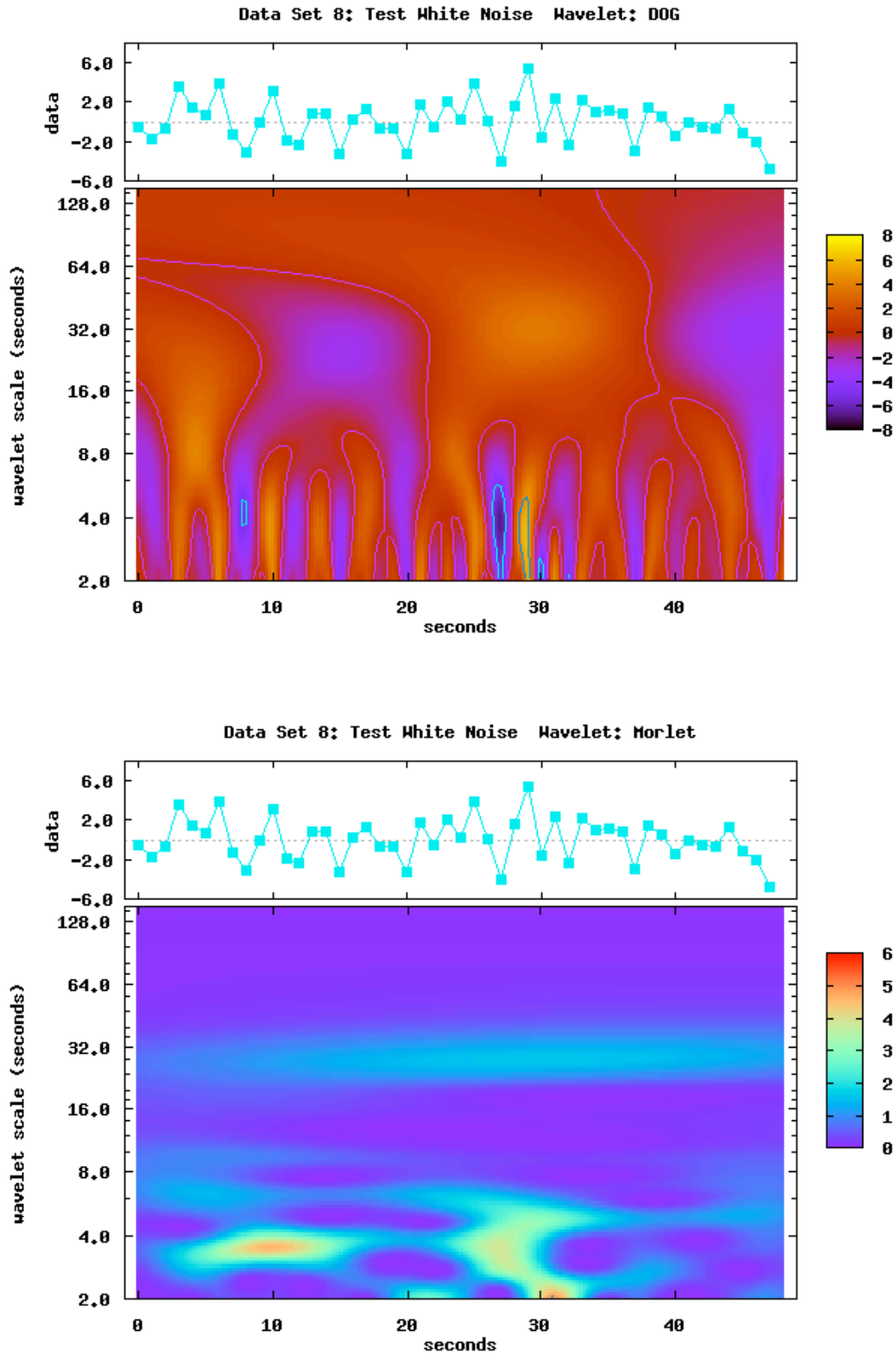
Figure 9. Wavelet transform scalograms for Gaussian white noise (σ=2.0). The same random seed was used to generate this set as for the noise added to Data Set 7 (see Fig. 8).